

Powerful relativistic electron beams with an energy of particles $E \sim 1$ MeV are considered as one possible source of energy for producing a thermonuclear plasma in forward systems. Energy can be transferred here to the plasma either directly by a powerful relativistic electron beam or through multicharge ions collectively accelerated in such a beam [1]. Of considerable interest in either case is information about changes in the energy distribution of electrons in the original beam. A strong magnetic field, usually also present in these systems, makes it difficult to analyze the energy distribution of electrons with the aid of an electric and a magnetic field. In view of this, at the Institute of Nuclear Physics (Siberian Branch of the USSR Academy of Sciences) a method is being developed for measuring the energy distribution of relativistic electrons with a nanosecond time resolution on the basis of the laws of electron retardation in a metal [2]. Two approaches are being considered. The first involves measuring in the metal the depth profile of absorbed electrons, the second involves measuring the electron current flowing to the collector through metal foils of various thicknesses. Both approaches require a subsequent retrieval of the electron energy distribution function from a Fredholm-type integral equation of the first kind.

Determination of the Energy Spectrum of Relativistic Electrons
from Their Depth Profile of Absorption in a Metal

The gist of this method is as follows. A beam of relativistic electrons impinges on a stack of n metal foils insulated from one another and is absorbed by them. The respective currents J_1, \dots, J_n of absorbed electrons are measured in the circuit of each foil. These currents are then used for calculating the electron energy distribution function.

The problem of determining the distribution function is solved as follows. We consider an electron beam normally incident on a plane metal surface. Let the function $K(x, E)$ describe the electron energy distribution function within the interval $[E_l, E_u]$ over the depth x , these electrons having been thermalized prior to their absorption from a monoenergetic beam with energy $E \in [E_l, E_u]$. For a beam with an arbitrary energy distribution of electrons $\varphi(E)$, where $E \in [E_l, E_u]$, the law of absorption in a metal can then be expressed as

$$dJ(x, E) = J_0 K(x, E) \varphi(E) dx dE, \quad (1)$$

with x denoting the distance normal to the surface from the latter to a given depth, $\varphi(E)$

denoting the electron energy distribution function for the original beam, $\int_{E_l}^{E_u} \varphi(E) dE = 1$;

dJ denoting the current of electrons with energies within the interval $(E, E + dE)$ absorbed within the distance $(x, x + dx)$, and J_0 denoting to total beam current.

We now subdivide a metal specimen into n foils of thicknesses $(x_{j+1} - x_j)$. It then follows from the law of absorption (1) that the j -th foil will absorb a fraction of the beam current equal to

$$f_j = \frac{J_j}{J_0} = \int_{x_j}^{x_{j+1}} dx \int_{E_l}^{E_u} K(x, E) \varphi(E) dE, \quad j = 1, \dots, n. \quad (2)$$

We next subdivide the energy interval $[E_l, E_u]$ into n segments of E_i ($i = 1, 2, \dots, n + 1$), where $E_1 = E_l$ and $E_{n+1} = E_u$. Replacing with a sum the integral with respect to energy in expression (2) leads to a linear system of equations

$$f_j = \sum_{i=1}^n K_{ji} \varphi_i, \quad j = 1, \dots, n, \quad (3)$$

where

$$K_{ji} = \int_{x_j}^{x_{j+1}} dx \int_{E_i}^{E_{i+1}} K(x, E) dE; \quad \varphi_i = \varphi(\tilde{E}_i); \quad \tilde{E}_i \in [E_i, E_{i+1}].$$

The solution to the system (3) is

$$\varphi_i = \sum_{j=1}^n M_{ij} f_j, \quad i = 1, \dots, n,$$

where M is the inverse of matrix K ($M = K^{-1}$).

Let us estimate the error of this solution. Inasmuch as the fraction f_j is determined experimentally from measurement of the currents in the foils, it obviously differs by some amount from the true value.

Let s_j be the rms error of an f_j determination and $s = \left(\prod_{j=1}^n s_j \right)^{1/n}$. We introduce the

notation $g_j = (s/s_j) f_j$, $L_{ji} = (s/s_j) K_{ji}$, $i, j = 1, \dots, n$.

Then system (3) becomes

$$\sum_{i=1}^n L_{ji} \varphi_i = g_j, \quad j = 1, \dots, n. \quad (4)$$

According to the results in [3], the error of determination of the i -th component of vector φ can be estimated as

$$\sigma_i^2 = s^2 [(L^*L)^{-1}]_{ii},$$

where $L_{ij}^* = L_{ji}$, and the mean-square error of the entire solution is

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n \sigma_i^2 = \frac{s^2}{n} \text{Sp} [(L^*L)^{-1}] = \frac{s^2}{n} \sum_{i=1}^n \frac{1}{\lambda_i^2}, \quad (5)$$

where λ_i^2 are the eigenvalues of matrix L^*L .

When all eigenvalues λ_i satisfy the condition $\lambda_i^2 \geq 1$, therefore, then the mean-square error of σ^2 of the solution does not exceed the experimental error s^2 . When even only one eigenvalue is $\lambda_i^2 \ll 1/n$, however, then the error of the retrieved solution becomes so large that the solution to system of equations (4) will be entirely meaningless.

Inasmuch as the elements of matrix L^*L and thus also its eigenvalues are determined by the points of subdivisions x_1, \dots, x_{n+1} and E_1, \dots, E_{n+1} , it must be possible to optimize these subdivisions. We select the scheme of subdivision which will result in the minimum value of $\sum_{i=1}^n \lambda_i^{-2}$. This will greatly reduce the error of retrieval of the electron energy distribution function $\varphi(E)$. The retrieval error will, however, still be rather large.

The error of retrieval of the distribution function could be further reduced by the method of statistical regularization, widely used for the solving of Fredholm integral equations of the first kind [3].

This method consists of replacing the exact equations in system (4) with approximate (regularized) ones. Such a replacement corresponds to introducing additional information about the solution by way of stipulating its a priori probability distribution. In our case this additional information can serve as an assumption about the boundedness of the electron energy distribution function $\varphi(E)$.

Let the approximate value of the norm of vector $\varphi(E)$ be given as

$$|\varphi| = \int_{E_l}^{E_u} \varphi^2(E) dE \approx \omega. \quad (6)$$

Representing the integral in expression (6) as a sum, we can obtain an explicit expression for the function of norm ω . Indeed, letting

$$|\varphi| = (\varphi, \Omega\varphi) \equiv \sum_{i,j=1}^n \varphi_i \Omega_{ij} \varphi_j \approx \omega,$$

we find that $\Omega_{ij} = \delta_{ij}(E_{i+1} - E_i)$.

As has been shown already [3], the regularized solution φ^α to system of equations (4) which satisfies condition (6) is the solution to system

$$(L^*L + \alpha s^2 \Omega)\varphi^\alpha = L^*g, \quad (7)$$

with the regularization parameter $\alpha = n/\omega$. Parameter α can be determined approximately, assuming a sufficiently smooth $\varphi(E)$ function and proceeding as follows. Condition (6) yields

$$\int_{E_l}^{E_u} \varphi^2(E) dE \approx \sum_{i=1}^n \frac{\varepsilon_i^2}{E_{i+1} - E_i} \approx \omega, \quad (8)$$

where $\varepsilon_i = \int_{E_i}^{E_{i+1}} \varphi(E) dE$.

Since $\sum_{i=1}^n \varepsilon_i = 1$ and, consequently, $0 \leq \varepsilon_i \leq 1$, expression (8) yields the range within which ω can vary:

$$\frac{1}{E_u - E_l} \leq \omega \leq \frac{1}{\min_i (E_{i+1} - E_i)}.$$

Therefore, the range of allowable values of the regularization parameter α is

$$n \min_i (E_{i+1} - E_i) \leq \alpha \leq n(E_u - E_l). \quad (9)$$

The error of retrieval of the i -th component of vector φ^α according to the system of equations (7) will be expressed as

$$\sigma_i^2 = s^2 [(L^*L + \alpha s^2 \Omega)^{-1}]_{ii}. \quad (10)$$

This level of error is sufficiently low so as to allow using the solution to system (7) as the electron energy distribution function.

The said procedure of determining the electron energy distribution function was implemented on a model BESM-6 high-speed computer. Matrix K_{ji} of the system of equations (3) was constructed with the aid of tables [4] of calculated data characterizing the distribution of thermalized electrons in aluminum corresponding to initial electron energies of 0.1, 0.3, 0.5, 0.7, and 1.3 MeV. The graph of function $K(x, E)$ plotted from these tables for energies of 0.1, 0.5, and 1.0 MeV is shown in Fig. 1 (points 1-3, respectively). We note that, upon normalization of the thickness x of the material with respect to the extrapolated mean free path $R_0(E)$, the $K(x, E)$ curves for various energies coincide within ~10% and, consequently, the function $K(x, E)$ can as accurately be represented by the function $K(\xi)$, where $\xi = x/R_0(E)$.

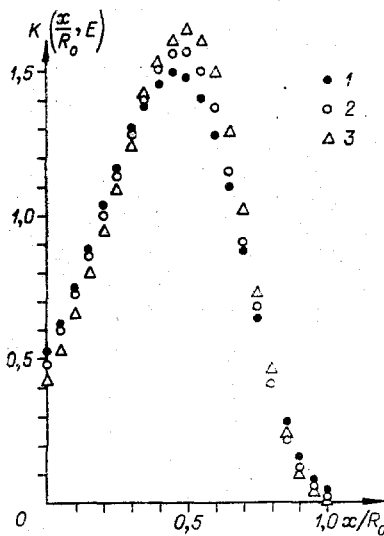


Fig. 1

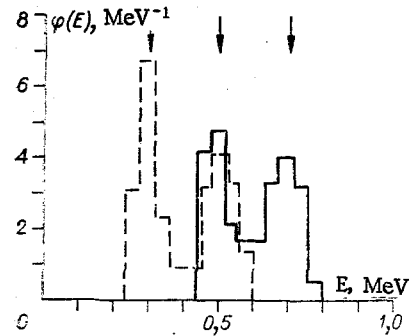


Fig. 2

TABLE 1

Foil No.	Foil thickness, μm	Current of electrons absorbed by the foils			
		0.3 and 0.5 MeV		0.5 and 0.7 MeV	
		J'	J''	J'	J''
1	112	0,172	0,171	0,072	0,071
2	83	0,208	0,208	0,078	0,077
3	89	0,210	0,206	0,105	0,106
4	100	0,130	0,132	0,142	0,142
5	150	0,132	0,128	0,219	0,218
6	200	0,065	0,065	0,190	0,191
7	225	$2 \cdot 10^{-3}$	$6 \cdot 10^{-3}$	0,095	0,094
8	355	0	$6 \cdot 10^{-5}$	0,023	0,024
9	5000	0	0	0	10^{-2}

Integration with respect to x , for constructing the K_{ji} matrix, was performed according to Simpson's rule in $10^{-2} (x_{j+1}/R_0(E) - x_j/R_0(E))$ steps for the j -th interval ($j = 1, \dots, 9$). The integral with respect to energy was replaced with the algebraic expression

$$K_{ji} = \frac{1}{2} [K_j(E_i) + K_j(E_{i+1})] (E_{i+1} - E_i), \quad i = 1, \dots, 9,$$

with

$$K_j(E_i) = \int_{x_j}^{x_{j+1}} \frac{dx}{R_0(E_i)} K(x, E_i), \quad j = 1, \dots, 9.$$

The values of function $K(x, E)$ at intermediate energy levels and intermediate coordinate points not listed in the tables [4] were calculated by quadratic interpolation. The value of $R_0(E)$ for aluminum was determined according to the approximation [5]

$$R_0(E) = 0.661E \left(1 - \frac{0.9878}{1 + 3.83E} \right), \quad (11)$$

with the energy of electrons E in MeV and the extrapolated mean free path R_0 in g/cm^2 .

The optimum subdivision into foil thicknesses and energy intervals, the latter within the 0.15–1.1-MeV range, was found as a result of computer-aided sifting of variants and search for the subdivision scheme with the minimum error of solution (5). On the resultant optimum system of linear equations we then performed a statistical regularization of its solution. This required determining the range of allowable values of the regularization parameter α according to expression (9) and then constructing the regularized system of equations (7).

The program was checked as follows. The currents in foils corresponding to a superposition of polyenergetic beams with a given energy were calculated according to Eqs. (2). These currents, with an rms error of 5-10%, were used as the input data for solving the system (7). The error of the resulting solution was estimated according to Eq. (10). The distribution function obtained in the process was compared to the original one. The form of the determined distribution was found to depend strongly on the value of the regularization parameter α . When $\alpha \approx n(E_u - E_l) \sim 9$ MeV, then the solution to system (7) depends weakly on E and approaches the constant value $\varphi(E) \approx 1/(E_u - E_l) \sim 1/9$ MeV⁻¹. As α is decreased, the solution approaches the original distribution function and the result based on $\alpha \approx n \min (E_{i+1} - E_i) \sim 0.45$ MeV becomes finally acceptable. In this case the currents in foils calculated by integration of the original distribution function (current J') closely approach the distribution function obtained in the process of solving system (7) (current J''). For comparison, in Table 1 the values of currents are given and in Fig. 2 are shown the distribution functions corresponding to an original distribution function selected as a superposition of two identical monoenergetic beams: 0.3 and 0.5 MeV in one example, 0.5 and 0.7 MeV in another. These energies have been indicated in Fig. 2 by arrows. The solution was sought as follows. First the entire energy range of 0.1-1.2 MeV was subdivided into nine intervals. A system of equations (7) was then constructed and the distribution function was calculated. Along the energy axis was demarcated only the range within which the calculated function differed from zero. For this range we again constructed and solved a system (7). The resolving power of system (7) can be greatly improved in this way. According to the diagram in Fig. 2, an energy resolution of ~ 0.10 MeV is attainable with only nine foils.

As a practical example of using this method, we will show measurements of the distribution function in a relativistic electron beam within a pulse of $\tau \sim 70$ -nsec duration after its passage through a plasma column, of a length $l \sim 2$ m and a density $n_0 \sim 3 \cdot 10^{13}$ cm⁻³, bounded at both ends by 50- μ m-thick titanium foils [6]. The scheme of measurements has already been thoroughly described in an earlier study [2]. The thicknesses of foils as well as the currents* J' of electrons absorbed by these foils and measured at two instants of time are given in Table 2. The voltage across the anode-cathode gap in the accelerator, at the instant of beam formation, was in those two cases 0.8 and 0.6 MV, respectively. From the current readings have been calculated distribution functions, histograms of which are shown in Fig. 3 (1 refers to 0.8 MeV, 2 refers to 0.6 MeV). It is evident here that the original beam (initial energy indicated by an arrow) loses a large fraction of its energy ($\sim 20\%$), which agrees with the results obtained by other methods of estimating its total losses [6].

Interestingly enough, a multifoil analyzer of the electron energy spectrum was used in another study [7] and there the energy distribution was determined without time resolution. The authenticity of the electron energy spectrum obtained was checked by comparing it with measurements of the energy distribution made with a magnetic analyzer. The results obtained by both these independent methods were found to be in close agreement. In that study [7] as well as in this study here the distribution function $K(x, E)$ of thermalized electrons was taken from [4].

Determination of the Energy Spectrum of Relativistic Electrons from the Electron Flux Attenuated by Aluminum Foils

The second approach to determining the energy of relativistic electrons is based on measuring the attenuation of the electron flux by metal foils of various thicknesses. The procedure for such an experiment is as follows. On a Faraday cylinder made of graphite impinges a beam of relativistic electrons. In the path of this beam, before the graphite cylinder, is placed an aluminum foil. As the thickness of this foil is increased, the current recorded by the Faraday cylinder decreases.

It has been shown experimentally [8] that such an attenuation of the beam by a metal is universal in nature in the case of a monochromatic beam of relativistic electrons within the $0.15 \leq E \leq 1$ -MeV energy range. This means that the electron transmission coefficient $K'(x, E)$ can be represented as a function of the single argument $\xi = x/R(E)$, with $R(E)$ denoting the depth of total absorption of electrons with energy E. The normalizing length $R(E)$ was calculated according to the expression for ionization losses [9]. We note that Eq. (11) [5]

*The accuracy of these measurements was determined by the error in calibrating the relative sensitivity of the recording channels (7%) and the error in reading the oscillograms (4%).

TABLE 2

Foil No.	Foil thickness, μm	Current of electrons absorbed by the foils			
		0.8 MeV		0.6 MeV	
		J'	J''	J'	J''
1	100	0,164	0,163	0,183	0,176
2	100	0,109	0,119	0,219	0,223
3	100	0,128	0,122	0,167	0,167
4	100	0,105	0,095	0,150	0,143
5	100	0,096	0,097	0,092	0,099
6	100	0,077	0,096	0,071	0,075
7	200	0,189	0,176	0,079	0,081
8	200	0,088	0,094	0,029	0,030
9	5000	0,044	0,043	0,010	0,010

Note. J'' are the currents in foils obtained by integrating, according to Eq. (2), the calculated distribution function.

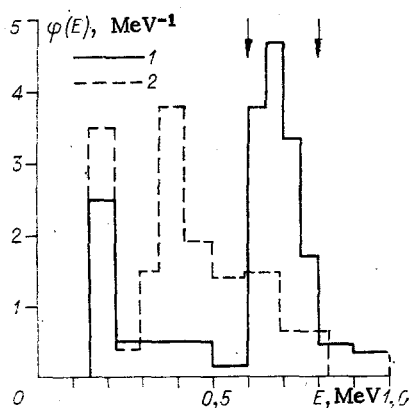


Fig. 3

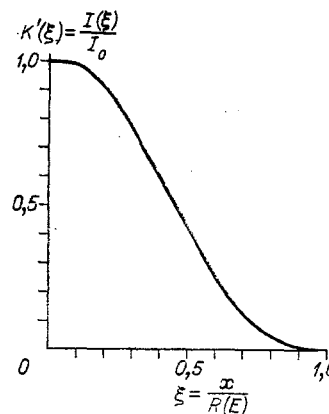


Fig. 4

for the depth of total absorption yields a result close to the result for the 0.03-5-MeV range [9] within a few percent. The universal curve of the transmission coefficient $K'(\xi)$ for aluminum is shown in Fig. 4.

The universality of $K'(\xi)$ was utilized in another study [10] for measuring the energy of electrons in a steady monoenergetic beam on the basis of their passage through a foil of given thickness x_1 . From the attenuation of the beam by the foil was, according to the curve in Fig. 4, determined the parameter ξ and from the latter the normalizing length $R = x_1/\xi$. The energy of particles was calculated according to the expression

$$E = 0.76R(1 + \sqrt{1 + 0.69/R}).$$

An analogous procedure for a nonsteady monoenergetic electron beam has been described in [11].

A natural outgrowth of this method is its extension to a nonmonoenergetic electron beam. With the beam parameters held invariant and the foil thickness changed, one performs n tests where current I_j into the Faraday cylinder is measured for each thickness x_j ($j = 1, \dots, n$), respectively.

The beam current into the Faraday cylinder after passage through a foil of thickness x_j is

$$I_j = I_0 \int_{E_l}^{E_u} \varphi(E) K' \left(\frac{x_j}{R(E)} \right) dE, \quad j = 1, \dots, n,$$

where $\varphi(E)$ denotes the electron energy distribution function. Let us subdivide the energy interval $[E_l, E_u]$ into n segments by levels E_i ($i = 1, \dots, n-1$). Assuming a sufficiently

smooth function $\varphi(E)$, one can replace the integral with a sum

$$f_j = \sum_{i=1}^n K'_{ji} \varphi_i, \quad j=1, \dots, n, \quad (12)$$

where

$$f_j = \frac{I_j}{I_0}; \quad K'_{ji} = \int_{E_{i-1}}^{E_i} K' \left(\frac{x_j}{R(E)} \right) dE \quad (i=1, \dots, n); \quad \varphi_i = \varphi(\tilde{E}_i); \quad \tilde{E}_i \in [E_{i-1}, E_i]; \quad E_0 = E_l; \quad E_n = E_u.$$

We thus obtain a system of linear equations analogous to system (3). System (12) is solved by the same methods as system (3).

We note that, in the case of a thick aluminum plate serving as electron collector, the data in [4] can be used for a numerical evaluation of the results,

This method was used in the INAR apparatus for retrieving the energy spectrum of relativistic electrons toward a beam. The experimental procedure has already been described [12]. The counterflux of electrons at a plasma density $n_0 \approx 3 \cdot 10^{13} \text{ cm}^{-3}$ was measured for various thicknesses of the attenuating foils (Fig. 5). The experimental points in Fig. 5 correspond to a time of 7 nsec after the beginning of a pulse of counteracting electrons. The standard deviation at each point does not exceed 7%. From the measured attenuation of the electron flux was then calculated the energy distribution of electrons (Fig. 6). The error of retrieval of this distribution is ~30%.

Obviously, this method is suitable only for experiments with good repeatability of results. One of its advantages is the possibility of improving the energy resolution appreciably by increasing the number of tests.

In the preceding considerations of the retrieval of the electron energy distribution there has not been touched upon the problem of the effect which angular spread of the electron beam entering a foil-type analyzer has on the results of calculations, namely on the energy distribution function. The scheme of calculations included the absorption law referring to electrons normally incident on a metal surface. This appears to be entirely justified, inasmuch as calculations of the absorption of electrons with a Gaussian angular distribution by aluminum have shown that only at angles $(\langle \theta^2 \rangle)^{1/2} \geq 20^\circ$ does the absorption law begin to depart appreciably (~10%) from the curves in Figs. 1 and 4. In order to use the data in [4] and [8] for calculations, therefore, it is necessary to ensure in the experiment that the angular spread of the electron beam at the analyzer entrance be $\leq 15^\circ$. In apparatus of the INAR type this is easily attained by adiabatic conversion of the transverse momentum of electrons to a longitudinal one in a tapering magnetic field at the exit from the "plugotron." We will mention a few more factors which must be considered in the design of a foil-type analyzer: the driving magnetic field in the apparatus, the intrinsic space charge of the beam, the mutual effect of adjacent foils on each other at high frequencies, and also the frequency band of the entire recording system.

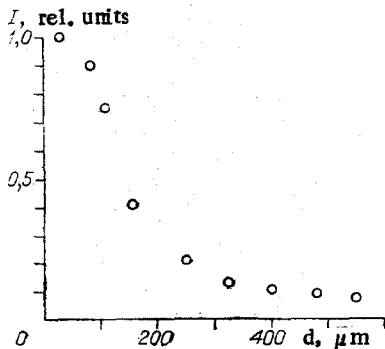


Fig. 5

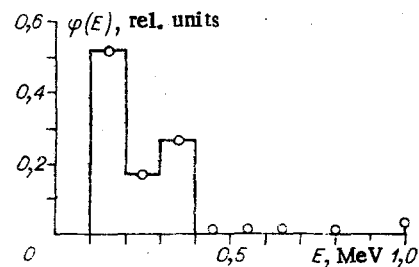


Fig. 6

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ELECTRIC FIELD INTENSITY DUE TO AN ARC IN A DEVELOPED TURBULENT AIR STREAM

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In plasmotrons with an interelectrode insert of sufficient relative length there evolve all three regions which characterize the flow of a gas through a tube (initial, transitional, and developed turbulent [1]). As is well known [1, 2], the electric field intensity of the arc differs in these regions. It is technically highest in the third channel region, much higher than in the initial one. Studies of the aerodynamics of gas flow through channels have contributed to the development of simple and highly efficient methods of controlling the length of the initial flow region and thus also that of the turbulent one, making it possible to intentionally influence the integral electrical characteristic of an arc — the voltage. In view of this, there has arisen the necessity to generalize the electrical characteristic of an arc glowing in a developed turbulent stream, i.e., to find how the electric field intensity of an arc in such a stream depends on the current, on the channel diameter, on the pressure, and on the gas flow rate. Such a generalization is important, moreover, because plasmotrons with an interelectrode insert are now the most promising devices of this kind, not only on account of the high power that can be pumped into the arc but also on account of the efficient conversion of electric energy to heat. Meanwhile, however, most theoretical and experimental studies have dealt mainly with arcs glowing in a laminar gas stream. In real plasmotrons of linear configuration with gas-vortex stabilization of the arc, on the other hand, the Reynolds number of the stream is usually higher than critical [3]. Several

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